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The fractional Aharonov–Bohm effect in mesoscopic rings

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Abstract. We study the effects of correlations on a one-dimensional ring threaded by a uniform magnetic flux. In order to describe the interaction between particles, we work in the framework of the $U = \infty$ Hubbard model and the t–J_z and t–J_z–J_t models. We focus on the dilute limit. Our results suggest the possibility that the persistent current has an anomalous periodicity ϕ_0/p, where p is an integer in the range $2 \leqslant p \leqslant N_e$ (N_e is the number of particles in the ring and ϕ_0 is the flux quantum). We find that this result depends neither on disorder nor on the detailed form of the interaction, while there is still on-site infinite repulsion.

1. Introduction

Persistent currents in metal rings enclosing a magnetic flux were first studied in the sixties [1]. In ideal systems the periodicity of the magnetoconductivity is given by the flux quantum $\phi_0 = hc/e$. In 1981, Al'tshuler, Aronov and Spivak [2] renewed the interest in the topic by predicting that in highly disordered cylinders the period of the magnetoconductivity was $\phi_0/2$. This effect was then confirmed experimentally by Sharvin and Sharvin [3]. Some years later, Büttiker and co-workers predicted persistent currents in a one-dimensional loop of normal metal driven by an external magnetic flux with elastic [4] and inelastic [5] scattering. In 1984, Gefen *et al* [6] studied the case with leads, obtaining a ϕ_0-periodicity in the conductivity.

The predicted current in multichannel normal-metal rings was analysed as a function of disorder and temperature [7] and it was found that disorder strongly reduces the amplitude of the persistent current [8].

In 1990, Lévy, Dolan, Dunsmuir and Bouchiat [9] found experimental evidence of these currents. In this experiment, the response of an ensemble of mesoscopic copper rings was measured as a function of the enclosed magnetic flux. They observed a periodicity of half a flux quantum. In the same year, Ambegaokar and Eckern [10] calculated the disorder-averaged current in an ensemble of rings including interactions between electrons and found a half-quantum-flux periodicity and that the amplitude of the persistent current was $\sim ev_F\eta/L^2$ (where L is the perimeter of the ring, v_F is the Fermi velocity and η is the elastic mean free path) in agreement with measurements [9]. The experiment of Lévy *et al* motivated a great deal of activity concerning the problem of persistent currents in an ensemble of rings [11]. In 1991, Chandrasekhar *et al* [12] measured the current in a single, isolated gold ring. In references [9] and [12] the rings were metallic, but in 1993 Mailly *et al* [13] reported measurement of the persistent current in a semiconductor ring. Recently, new experiments were made on those rings [14] emphasizing the differences between isolated and connected geometries.

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For a multichannel single ring with impurities and in the diffusive regime, the magnitude of the current is expected to be $\sim ev_F\eta/L^2$ [8]. However, in an isolated ring [12] a persistent current one to two orders of magnitude larger than that predicted by theory was measured. This fact has stimulated many recent theoretical works with controversial results [15–19].

The aim of this work is to present results in the limit of strong coupling and low carrier density which are independent of the properties of the material. The paper is organized as follows. In section 2 we present the model and discuss its different limits. In section 3 we show the corresponding results and, finally, in section 4 we give their physical interpretation and the conclusions.

2. The model

We study a strictly one-dimensional ring threaded by a magnetic field. This is such that it produces a flux ϕ concatenated by the ring. In this system the ground state carries a steady current, which is periodic in the magnetic flux threading the loop, with period $\phi_0 = he/c$, the flux quantum. The current arises from the boundary conditions imposed by the magnetic flux.

To study the properties of a system of interacting electrons in a ring with N_s sites we use the model proposed by Hubbard [20]. The Hamiltonian of the extended Hubbard model is

$$\hat{H}_e = \hat{H}_U + \sum_{i=1}^{N_s} \epsilon_i \,(\hat{n}_{i,\uparrow} + \hat{n}_{i,\downarrow}) + \sum_{\substack{i=1 \\ \sigma,\sigma'=\uparrow,\downarrow}}^{N_s} \sum_{m=1}^{r_0} V_m \,\hat{n}_{i,\sigma}\hat{n}_{i+m,\sigma'} \tag{1}$$

where $\hat{H}_U$ is the Hubbard Hamiltonian:

$$\hat{H}_U = -t\left(\sum_{\substack{i=1 \\ \sigma=\uparrow,\downarrow}}^{N_s-1} \hat{c}^{\dagger}_{i,\sigma}\,\hat{c}_{i+1,\sigma} + \sum_{\sigma=\uparrow,\downarrow} \mathrm{e}^{\mathrm{i}2\pi\phi/\phi_0}\hat{c}^{\dagger}_{N_s,\sigma}\hat{c}_{1,\sigma}\right) + \mathrm{HC} + U\sum_{i=1}^{N_s} \hat{n}_{i,\uparrow}\,\hat{n}_{i,\downarrow} \tag{2}$$

and $\hat{c}_{i,\sigma}$ ($\hat{c}^{\dagger}_{i,\sigma}$) are the annihilation (creation) operators which annihilate (create) an electron of spin σ on a site i of the ring, $\hat{n}_{i,\sigma} = \hat{c}^{\dagger}_{i,\sigma}\hat{c}_{i,\sigma}$ is the number operator that counts the number of electrons of spin σ on the site i, and N_s is the number of sites in the chain.

The first term represents the kinetic energy describing the hopping of an electron from one site to a nearest-neighbour site with hopping matrix element t. We shall put $t = 1$ throughout this paper, thus fixing the energy scale.

The energies ϵ_i represent the disorder in the ring, and they can be any number in the range $[-W/2, W/2]$ with equal probability.

The last term allows us to represent a long-range contribution (r_0 gives the extent) where V_m is the strength of the interaction. When this term is zero, we have that the long-range contribution due to the Coulomb interaction is supposed to be screened, and is only retained when the electrons are in the same site giving an additional energy U when the site is doubly occupied.

Here we will consider neither the interaction through phonons nor any other solid excitation and therefore we restrict ourselves to the case where $U \geqslant 0$.

In the case of the Hubbard model with $U = 0$, the ground state consists of doubly occupied levels filling up to the Fermi level. In this state the total spin is the lowest possible. When $U \neq 0$ the Coulomb repulsion tends to reduce double occupancy, increasing the total spin of the ground state. In this way, the Coulomb energy is reduced by a quantity proportional to U but the kinetic energy is enhanced. Then, it is found that the ground

state is determined by a competition between the Coulomb and kinetic energies. In one dimension and with adequate boundary conditions it is possible to demonstrate analytically [21] that the ground state always has the minimum total spin. In our case, where the system is finite and with periodic boundary conditions, we will see that the result depends on the number of particles in the ring.

On the other hand we consider the extreme case in which the intrasite interaction is infinite. The $U = \infty$ limit is obtained by projecting onto a subspace where double occupancy is not allowed. The corresponding projection operator $\hat{P}$ is

$$\hat{P} = \prod_{i=1}^{N_s} (\hat{I} - \hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow}). \tag{3}$$

Therefore, the Hamiltonian of the one-dimensional $U = \infty$ Hubbard model is

$$\hat{H}_\infty = -t\,\hat{P} \sum_{\sigma,i=1}^{N_s} \hat{c}^\dagger_{i,\sigma}\hat{c}_{i+1,\sigma}\,\hat{P}. \tag{4}$$

This restriction drastically reduces the size of the Hilbert space explored from a given *ansatz* by the action of $\hat{H}_\infty$. In one dimension there is another restriction on the size of the available Hilbert space. It comes from the fact that the relative positions of particles having different spins must be conserved (up to cyclic permutations) under the action of $\hat{H}_\infty$.

Another possibility for studying the large-U limit is given by the t–J model [22] which allows us to work within the same Hilbert subspace as for the $U = \infty$ case. The Hamiltonian of the t–J model is a particular case of the t–J_t–J_z model:

$$\hat{H}_{t-J_t-J_z} = \underbrace{\hat{H}_\infty}_{(a)} + J_t \sum_{i=1}^{N_s} \underbrace{S_i^+ S_{i+1}^-}_{(b)} + J_z \sum_{i=1}^{N_s} \underbrace{S_i^z S_{i+1}^z}_{(c)} + \text{CC} \tag{5}$$

when $J_t = J_z = 2t^2/U$.

The terms (b) and (c) have a different effect when they act on a state: the term (b) connects states having a different spin order on the chain, and the term (c), being diagonal, connects a state with itself. They will be discussed in the following sections. In this way, we can study the problem for large U perturbatively beginning from the $U = \infty$ Hubbard model.

3. Results

In this section we show exact results for the Hubbard and t–J Hamiltonian which were obtained by solving small-size systems exactly with the Lanczos algorithm [23, 24, 25]. We present also analytical results in the limit where $U = \infty$.

The problem of the Lanczos method is that in its implementation it is necessary to deal with vectors of length equal to the size of the Hilbert space under consideration. Since this size usually increases exponentially with the number of sites of the lattice, the technique is restricted to small chains. This method allows us to obtain the ground state, and its energy $E(\phi)$, and to compute the corresponding current.

In this paper we will concentrate on ground-state properties only, so we will assume that the energy level separation is in general much larger than the thermal energy. In this case the total current can be obtained through $j = -\partial E/\partial\phi$.

3.1. The Hubbard model with finite U

The persistent current of the non-interacting ring ($U = 0$) can be understood if we assume that the one-particle energy levels move along the free-particle dispersion relation as the magnetic field increases, reducing the energy difference between the levels that correspond to the wave vectors $-j/N_s$ and $(j-1)/N_s$ (where j is an integer). Therefore, if the system possesses $2n$ particles there is an accidental degeneracy between the states with $S = 0$ and $S = 1$ (S is the total spin of the many-body state). This occurs at a magnetic flux $\phi_c = \phi_0/2$ in the case where n is an odd integer or at $\phi_c = 0$ when n is an even integer. The current behaves monotonically as a function of the flux, having a discontinuity at ϕ_c. The Fourier spectrum of this function has components which correspond to periodicities ϕ_0/m (where m is an integer), the most important of them being at $m = 1$. The Coulomb interaction shifts these accidental degeneracies to other values of ϕ_c: $0 < \phi_c < \phi_0/2$. At this point there is a transition to an $S = 1$ ground state, because the Coulomb energy gained is greater than the kinetic energy lost by increasing the total spin. This transition can also be seen in the energy curve as a sharp peak at the transition point.

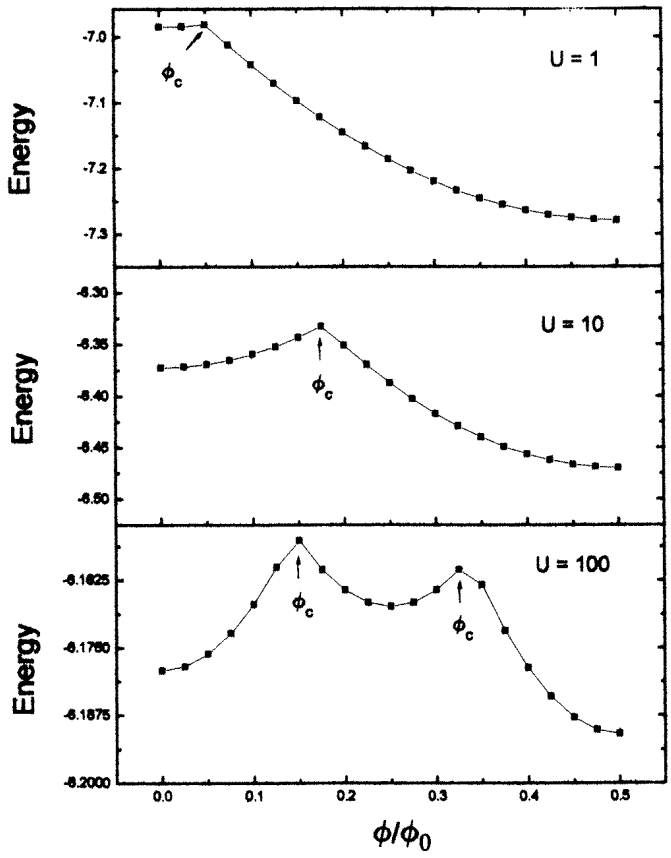

Figure 1. Energy versus the magnetic flux for four particles in a ten-sites ring, obtained using the Lanczos method. For $U = 1$ it is observed that a $\phi_0/2$ periodicity begins to appear in the energy curve. For greater values of U, the system tends to a $\phi_0/4$ periodicity.

If U is large enough, transitions to states with $S > 1$ are also possible. In any case, the number of peaks in the energy curve is increased; then, the Fourier components with $m > 1$ are enhanced. This is shown in figure 1.

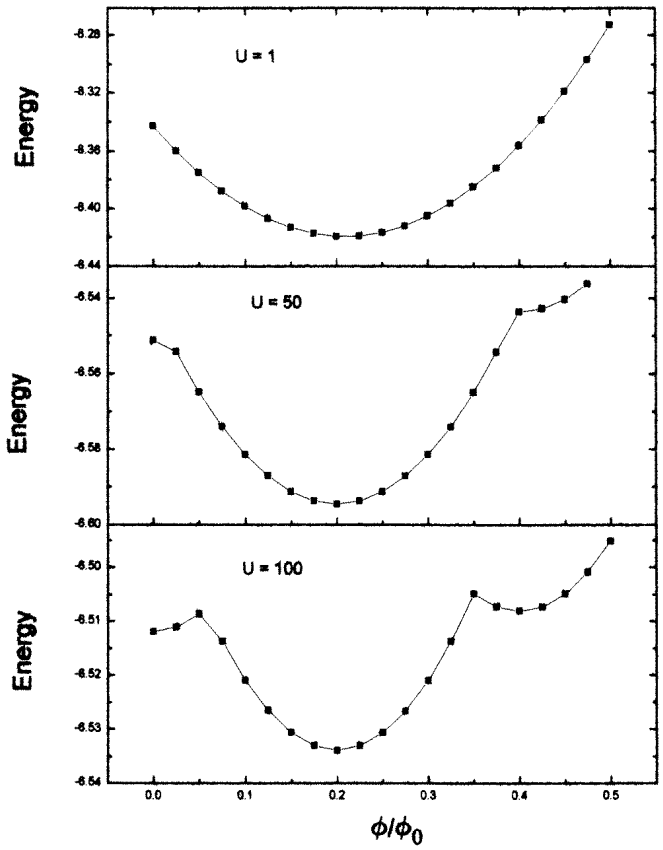

Figure 2. Energy versus the magnetic flux for five particles in a ten-sites ring, obtained using the Lanczos method. For $U = 1$ there are no peaks in the curve. Peaks appear for $U \sim 50$, and in the large-U limit the system tends to a $\phi_0/5$ periodicity.

If the number of particles in the ring is an odd number and the flux is close to the accidental degeneracies, there is no change of the total spin. This is so because a change of the total spin would imply filling levels of higher kinetic energy than in the even-particle case (due to the Pauli exclusion principle). So, at least for the smaller values of U, there are no peaks in the energy curve. Nevertheless, the transitions will occur at large values of U, large enough so as to compensate the kinetic energy increase. This is shown in figure 2 where we can see that the peaks appear at values of U greater than those of the even-particle case (see figure 1).

In the case of the extended Hubbard model, the main contribution comes from the disorder terms. The effect on these properties is twofold: it breaks translation symmetry and favours states with minimum total spin (the lower-energy sites on the chain tend to be doubly occupied). So, in this case there are neither accidental degeneracies nor transitions; therefore disorder tends to reduce the amplitude of the Fourier components with $m > 1$.

In order to study the limit of large U, a natural way is to consider the situation in which $U = \infty$, as we will see in the next section.

3.2. The $U = \infty$ Hubbard model

Here, we review the $U = \infty$ Hubbard model which provides a natural scenario for studying the t–J model.

It is well known that infinite one-dimensional systems present charge and spin separation. In other words we can say that the movement of holes leaves the relative spin order in the chain unaltered. Then, the Hamiltonian has a block form, each block corresponding to a definite spin order. However, when the system is finite and closed, a particle going from the last to the first site of the loop modifies the spin arrangement by a cyclic permutation. Even in that case the Hamiltonian has a block form, each block corresponding to a subspace where the spin order of the states can only differ by cyclic permutations of the particles. These subspaces can be labelled by an integer number F, which is the minimum number of cyclic permutations that must be performed in order to reobtain the initial state. For example, when we have four particles there are only two subspaces. They are

$$F_1 = 4 \longrightarrow \{|\uparrow\uparrow\downarrow\downarrow\rangle; |\uparrow\downarrow\downarrow\uparrow\rangle; |\downarrow\downarrow\uparrow\uparrow\rangle; |\downarrow\uparrow\uparrow\downarrow\rangle\}$$
$$F_2 = 2 \longrightarrow \{|\uparrow\downarrow\uparrow\downarrow\rangle; |\downarrow\uparrow\downarrow\uparrow\rangle\}$$

(empty sites are irrelevant for this analysis). Clearly,

$$\langle F_i|\hat{H}_\infty|F_j\rangle = 0 \qquad \text{if } F_i \neq F_j. \tag{6}$$

For a general number of particles N_e, it is possible to show that $2 \leqslant F_i \leqslant N_e$. In particular if N_e is an odd number, the only possibility is $F = N_e$.

Within each of these subspaces two states that differ by a cyclic permutation are connected by the boundary terms of the Hamiltonian

$$\hat{H}_b = -t\,\mathrm{e}^{\mathrm{i}2\pi\phi/\phi_0}\hat{c}^\dagger_{N_s,\sigma}\hat{c}_{1,\sigma}. \tag{7}$$

In the general case, the Hamiltonian within each F-subspace ($\hat{H}^F$) can be written as

$$\hat{H}^F = \begin{pmatrix} \hat{H}^{F,1} & \hat{T}\,\mathrm{e}^{(-\mathrm{i}2\pi\phi/\phi_0)} & \cdots & \hat{T}\,\mathrm{e}^{(-\mathrm{i}2\pi\phi/\phi_0)} \\ \hat{T}\,\mathrm{e}^{(\mathrm{i}2\pi\phi/\phi_0)} & \hat{H}^{F,2} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \hat{T}\,\mathrm{e}^{(\mathrm{i}2\pi\phi/\phi_0)} & 0 & \hat{T}\,\mathrm{e}^{(\mathrm{i}2\pi\phi/\phi_0)} & \hat{H}^{F,F} \end{pmatrix}. \tag{8}$$

Each matrix $\hat{H}^{F,j}$ is equivalent to a Hamiltonian of N_e spinless fermions on a chain with open boundary conditions associated with eigenvalues E_0^α and eigenvectors $|j, N_e, \alpha\rangle$.

In terms of this basis of eigenstates the Hamiltonian reads

$$\hat{H}^F = \sum_{j,\alpha} E_0^\alpha |j, N_e, \alpha\rangle\langle j, N_e, \alpha| + \sum_{j,\alpha,\beta} t'_{\alpha\beta}|j, N_e, \alpha\rangle\langle j+1, N_e, \beta|. \tag{9}$$

Each term in the first sum describes the matrix $H^{F,j}$. The second sum describes the matrix $\hat{T}\,\mathrm{e}^{(\mathrm{i}2\pi\phi/\phi_0)}$, where $t'_{\alpha\beta}$ is given by

$$t'_{\alpha\beta} = \langle j, N_e, \alpha|\hat{H}_b|j+1, N_e, \beta\rangle. \tag{10}$$

Because of the structure of the $H^{F,j}$-matrices, $t'_{\alpha\beta}$ does not depend on the indices j and F. Now, by using

$$|j, N_e, \alpha\rangle = \frac{1}{\sqrt{F}} \sum_p \mathrm{e}^{\mathrm{i}pj}|p, N_e, \alpha\rangle \tag{11}$$

where $p = (2\pi/F)n$ and n is an integer ($0 \leqslant n < F$), we find that the Hamiltonian can be written as

$$\hat{H}^F = \sum_p \left(\sum_\alpha E_0^\alpha |p, N_e, \alpha\rangle\langle p, N_e, \alpha| + \cos\left(p + 2\pi \frac{\phi}{\phi_0} \right) \sum_{\alpha,\beta} 2t_{\alpha\beta} |p, N_e, \alpha\rangle\langle p, N_e, \beta| \right)$$
$$= \sum_p H_p. \tag{12}$$

Three conclusions can be drawn from this formula.

(i) The energy of any eigenstate of H_p can be written as

$$E_0 + t' \cos\left(p + 2\pi \frac{\phi}{\phi_0} \right) \tag{13}$$

with E_0 and t' appropriate constants related to the first and second terms of (12). This formula gives the dependence of the many-body levels on the external flux.

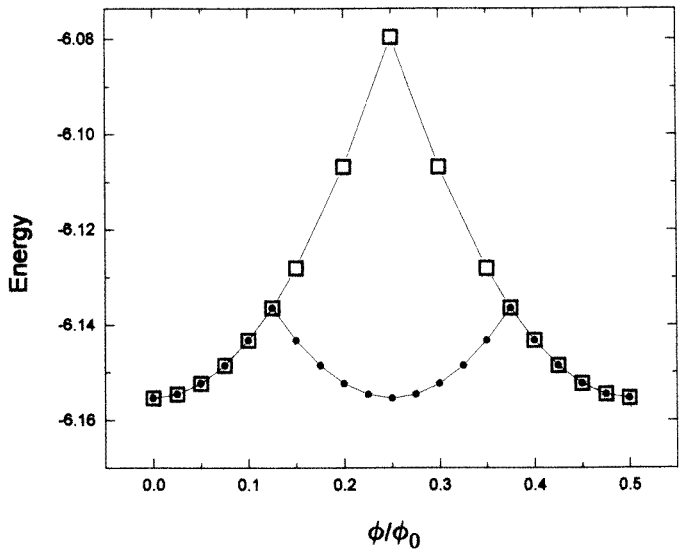

Figure 3. The ground-state energy as a function of the flux for four particles in a ten-sites ring obtained using the Lanczos method for the $U = \infty$ Hubbard model. Open squares indicate the energy in the $F = 2$ subspace, and solid circles indicate the energy in the $F = 4$ subspace. A continuous line is used to plot equation (13).

(ii) As the flux is increased, there are crossings between the many-body levels that correspond to different values of p, as happens in the free-particle case. In addition, the largest number of crossings occurs for the largest F-value. Therefore, we can say that the ground state always belongs to the subspace with maximum F, though it is possibly degenerate with others of lower F. This degeneracy does not exist over the whole range of flux (from 0 to ϕ_0). In every case, there is a region in which none of the F-subspaces are degenerate, i.e. there are energy gaps between them. This result is illustrated in figure 3 with a numerical example corresponding to four particles in a ten-sites ring. For this case, the energy within each F-subspace was evaluated beginning the Lanczos procedure with a configuration corresponding to the desired subspace. For example, we use $|\uparrow\uparrow\downarrow\downarrow\rangle$ for $F = 4$, and $|\downarrow\uparrow\downarrow\uparrow\rangle$ for $F = 2$.

(iii) From (8) we see that within every F-subspace the problem can be mapped onto a tight-binding chain with a magnetic flux $F\phi$. Therefore, the periodicity of the persistent current is ϕ_0/F. In particular, the ground-state periodicity is ϕ_0/N_e as was already stated in [26, 27]. But we are showing that anomalous flux quantization occurs also in the excited subspaces of the problem, corresponding to the periodicity ϕ_0/F (remember that $2 \leqslant F \leqslant N_e$). Note that in the case where we have an odd number of particles the only possibility is $F = N_e$ which leads to a persistent current having a periodicity ϕ_0/N_e in each subspace.

(iv) The periodicity of persistent current is given by the F-number of the state under consideration. States with $2 < F < N_e$ can be obtained also from spin-polarized configurations. For example, for eight particles, F can be 2, 4, 8 and we have states such as

$$F = 4 \longrightarrow \{|\uparrow\uparrow\uparrow\downarrow\uparrow\uparrow\uparrow\downarrow\rangle\}$$

which have four different cyclic permutations, i.e. belongs to an $F = 4$ subspace. These states can be favoured by an external magnetic field, and in this case a fractional periodicity with $2 < F < N_e$ occurs in the ground state of the system. A similar result was stated also in [28], but using different methods.

We would like to point out that our analysis of the periodicity of the persistent current is completely general and cannot be modified by including diagonal disorder or long-range interactions (while the double occupancy of the sites remains forbidden), which only affects the coefficients E_0 and $t_{\alpha\beta}$ and therefore only the amplitude of the persistent current. In each F-subspace, these coefficients are renormalized in the same way (if the interaction does not depend on the spin of the particles), so they cannot change the crossings between many-body levels, driven by the magnetic flux, and the fundamental periodicity remains unaltered.

3.3. The t–J model

Here we consider the effect of a large but finite U by means of the t–J model.

As an initial step we analyse the t–J_z model, obtained by setting $J_t = 0$ in equation (5). The Hamiltonian of this model does not connect states with different F-values. We have seen that the term (a) favours the maximum value for F, which corresponds to the minimum number of antiferromagnetic links. The term (c) favours the minimum value for F, which corresponds to the maximum number of antiferromagnetic links. This competition between (c) and (a) selects the F-subspace to which the ground state belongs, modifying the periodicity of the energy and the persistent current, as we show in figure 4. The effect associated with the AFC is to renormalize the eigenvalues E_0^α and the $t_{\alpha,\beta}$-coefficients. This renormalization is different within each F-subspace (due to the different number of antiferromagnetic links within each one), modifying the positions of the crossings. The most important harmonic of the energy curve as function of flux is defined by the lower value of F (see figure 4). Note that a periodicity of exactly a half of a quantum flux is readily seen as the antiferromagnetic coupling (AFC) is raised, because the subspace with $F = 2$ is stabilized over the whole range of flux. This suggests the possibility of superconducting correlations in the model. This is an important point: we present a model with a fundamental periodicity of a half of a quantum flux originated only by interactions. This result does not depend on the disorder, because it affects all of the F-subspaces in the same way.

When we include the term (b), the Hamiltonian connects states that belong to different F-subspaces. The effect is to change the spin arrangement. $J_t \ll 1$ means that the charact-

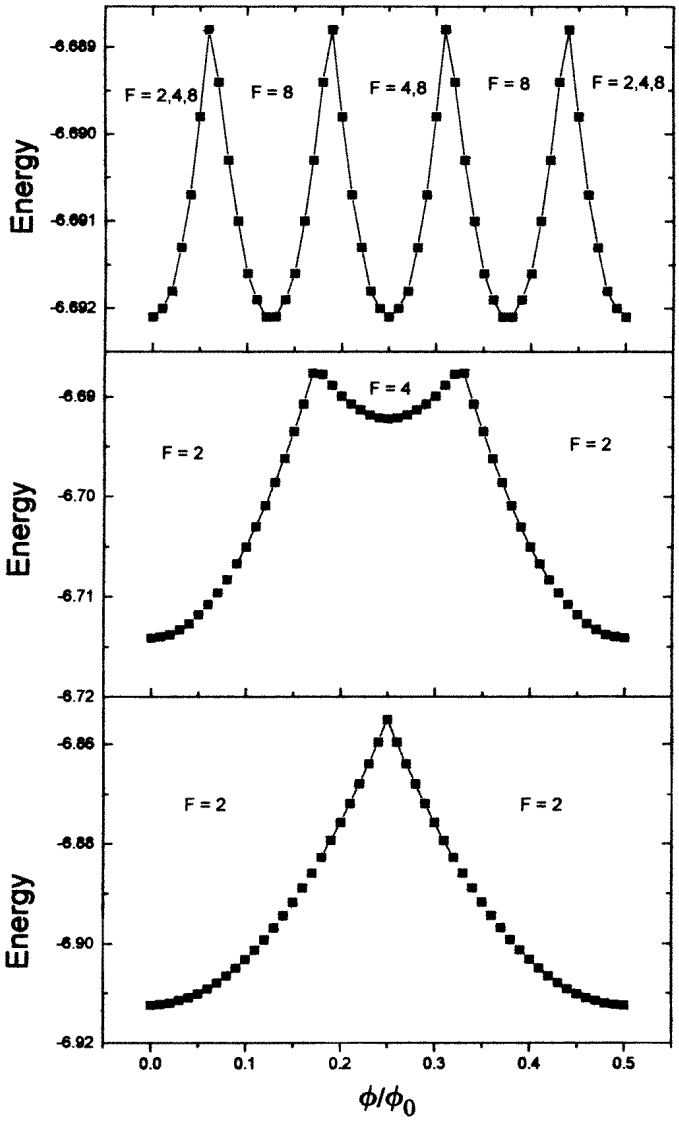

Figure 4. Energy versus the magnetic flux obtained by solving exactly using the Lanczos method the t–J_z model with eight particles in a twelve-sites ring for three values of J_z. From top to bottom the figures correspond to $J_z = 0, 0.01, 0.1$. In each flux interval between sharp peaks, the ground state corresponds to the numbers F indicated in the graph.

eristic time for a change in the spin arrangement is much greater than that corresponding to one-hole hoppping, so the charge can hop while the spin arrangement remains almost unaltered. Therefore in this limit the periodicity must be the same as in the t–J_z model. The only effect in this case is the smoothing of the current discontinuities.

For a strong enough J_t, or equivalently for non-dilute systems, the spin arrangement changes very quickly; then, when the charge hops, the phase coherence within each F-subspace disappears. In this case, the system recovers a periodicity of one quantum flux.

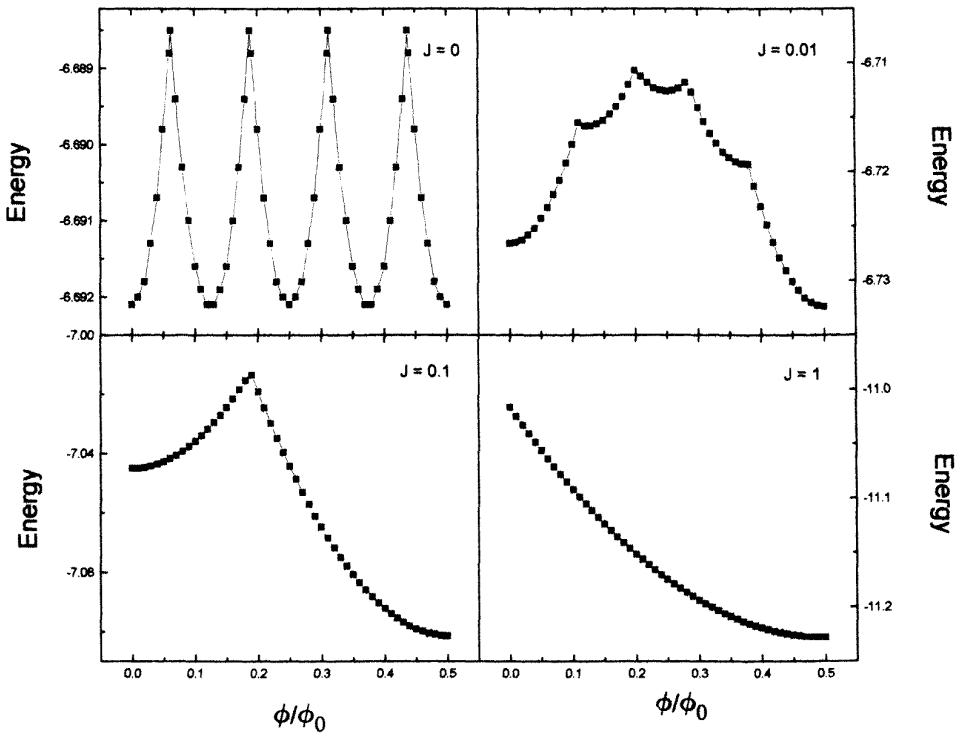

Figure 5. Energy versus the magnetic flux obtained by solving exactly using the Lanczos method the t–J model with eight particles in a twelve-sites ring for four values of J.

This is shown in figure 5. Note that even in this case there is a wide region of values of the AFC where the periodicity corresponds to a half of a quantum flux.

4. Conclusions

In the present work we have made a full study of a one-dimensional ring with infinite on-site repulsion and threaded by a magnetic flux.

In these systems we predict the existence of states which show a fractional Aharanov–Bohm effect (FABS) with periodicity ϕ_0/F, where F is an integer in the range $2 \leqslant F \leqslant N_e$. It is also suggested that a ground state with fractional periodicity different from ϕ_0/N_e could be stabilized due to Zeeman interaction [28]. Also we have shown that in the t–J_z model the AFC stabilizes a ground state which belongs to the $F = 2$ subspace over the whole range of flux and therefore with a periodicity of half of a quantum flux, suggesting the existence of superconducting correlations in the model. Fractional periodicities come from the crossing of many-body levels driven by the magnetic flux as shown in (13). These crossings are associated with different values of the quantum number p in equation (11). This symmetry is not broken by diagonal disorder, so disorder can neither modify the fractional periodicities nor round off cusps in the energy curve. Therefore the current amplitude (related to the derivative of the energy near to the cusp points) is essentially not modified by a weak disorder. However, due to the small period, the current has a small amplitude $\sim v_f/(LF)$,

where L is the length of the ring and F the integer labelling the states. This current will be suppressed only by strong enough disorder, when the energy-flux band becomes flat. Remarkably, the behaviour of the periodicity will be also independent of the detailed form of the interaction, while the on-site repulsion remains infinite. Spin fluctuations in the t–J_z–J_t model act as a relaxation mechanism for the FABS. They round off cusps in the energy curve and finally, if they are strong enough, they drive the system to a normal quantum flux periodicity.

To test our predictions it is necessary to set up an experiment where certain conditions must be fulfilled. On the one hand, interactions must be strong. On the other hand, it is important to work in the dilute regime because the spin fluctuations mentioned above can suppress the fractional periodicity. In addition, the system must be one dimensional. If Zeeman interactions are important, fractional periodicities with $2 \leqslant F \leqslant N_e$ could be present. However, it must be noted that our results predict a fractional periodicity $\phi_0/2$ even in the low-magnetic-field case, due only to electron–electron interactions. Nowadays it seems possible to construct systems such as quantum dot rings [29, 30] which could be an ideal tool for testing these properties.

Acknowledgments

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References

[1] Byers N and Yang C N 1961 *Phys. Rev. Lett.* **7** 46
Bloch F 1968 *Phys. Rev.* **166** 415
Schick M 1968 *Phys. Rev.* **166** 404
Gunther L and Imry Y 1969 *Solid State Commun.* **7** 1391
[2] Al'tshuler B L, Aronov A G and Spivak B Z 1981 *Pis. Zh. Eksp. Teor. Fiz.* **33** 101 (Engl. Transl. 1981 *JETP Lett.* **33** 94)
[3] Sharvin D Yu and Sharvin Yu V 1981 *Pis. Zh. Eksp. Teor. Fiz.* **34** 28 (Engl. Transl. 1981 *JETP Lett.* **34** 272)
[4] Büttiker M, Imry Y and Landauer R 1983 *Phys. Lett.* **96A** 365
[5] Landauer R and Büttiker M 1985 *Phys. Rev. Lett.* **54** 2049
[6] Gefen Y, Imry Y and Azbel M Ya 1984 *Phys. Rev. Lett.* **52** 129
[7] Cheung H, Gefen Y, Riedel E and Shih W 1988 *Phys. Rev.* **37** 6050
Bouchiat H and Montambaux G 1989 *J. Physique* **50** 2695
[8] Cheung H F, Riedel E K and Gefen Y 1989 *Phys. Rev. Lett.* **62** 587
[9] Lévy L, Dolan G, Dunsmuir J and Bouchiat H 1990 *Phys. Rev. Lett.* **64** 2074
[10] Ambegaokar V and Eckern U 1990 *Phys. Rev. Lett.* **65** 381
[11] von Oppen F and Riedel E 1991 *Phys. Rev. Lett.* **66** 84
Altshuler B L, Gefen Y and Imry Y 1991 *Phys. Rev. Lett.* **66** 88
[12] Chandrasekhar V, Webb R, Brady M, Ketchen M, Gallagher W and Kleinsasser A 1991 *Phys. Rev. Lett.* **67** 3578
[13] Mailly D, Chapelier C and Benoit A 1993 *Phys. Rev. Lett.* **70** 2020
[14] Reulet B, Ramin M, Bouchiat H and Mailly D 1995 *Phys. Rev. Lett.* **75** 124
[15] Abraham M and Berkovits R 1993 *Phys. Rev. Lett.* **70** 1509
[16] Fabrizio M, Gogolin A and Scheidl S 1994 *Phys. Rev. Lett.* **72** 2235
[17] Vignale G 1994 *Phys. Rev.* B **50** 7668
[18] Müller-Groeling A and Weidenmüller H A 1994 *Phys. Rev.* B **49** 4752
[19] Kato H and Yoshioka D 1994 *Phys. Rev.* B **50** 4943

[20] Hubbard J 1963 *Proc. R. Soc.* A **276** 238; 1964 *Proc. R. Soc.* A **277** 237
[21] Lieb E and Mattis D 1962 *Phys. Rev.* **125** 164
[22] Fradkin E 1991 *Field Theories of Condensed Matter Systems (Frontiers in Physics)* (Reading, MA: Addison-Wesley)
[23] Dagotto E 1994 *Rev. Mod. Phys.* **66** 763
[24] Gagliano E, Dagotto E, Moreo A and Alcaraz F 1986 *Phys. Rev.* B **34** 1677
[25] Dagotto E and Moreo A 1985 *Phys. Rev.* D **31** 865
[26] Kusmartsev F V 1991 *J. Phys.: Condens. Matter* **3** 3199
[27] Schofield A, Wheatley J and Xiang T 1991 *Phys. Rev.* B **44** 8349
[28] Kusmartsev F V 1995 *Phys. Rev.* B **52** 14 445
[29] Kirczenow G, Sachrajda A S, Feng Y, Taylor R P, Henning L, Wang J, Zawadski P and Coleridge P T 1994 *Phys. Rev. Lett.* **72** 2069
[30] Waugh F R, Berry M J, Mar D J, Westervelt R M, Campman K L and Gossard A C 1995 *Phys. Rev. Lett.* **75** 705